تکلیف [illegible] سوم تئوری ورق و پوسته

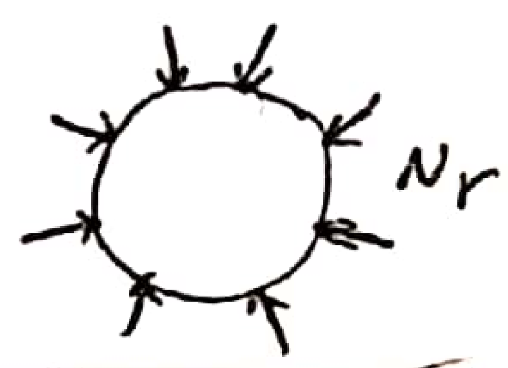
جواب سوال 1)

مختصات کارتزین

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left\{ (\nabla^2 w)^2 - 2(1-\nu)\left[w_{,xx} w_{,yy} - (w_{,xy})^2 \right] \right\} dxdy$$

$$-\int_0^a\int_0^b p w \, dxdy + \int_0^a\int_0^b \left[\frac{1}{2} N_x (w_{,x})^2 + \frac{1}{2} N_y (w_{,y})^2 + N_{xy} w_{,x} w_{,y} \right] dxdy$$

از همان نیروی محیطی صرف نظر شده است!



مختصات قطبی

$$\Pi = \frac{D}{2}\int_A \Big[\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right)^2 - 2(1-\nu)\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right)$$

$$+ 2(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta} \right)^2 \Big] \times 2\pi r dr d\theta$$

$$-\int_A p w \times 2\pi r dr d\theta + \int_A \Big[\frac{1}{2} N_x \times \left(\frac{\partial w}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial w}{\partial \theta}\sin\theta \right)^2 + \underbrace{\cdots}_{\text{بقیه ترم‌ها صفر هستند!}} \Big] \times 2\pi r dr d\theta$$

حالت متقارن

$$\frac{\partial}{\partial \theta} = 0 \longrightarrow \Pi = \pi D \iint_A \left[\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} \right)^2 - 2(1-\nu)\frac{d^2 w}{dr^2} \times \frac{1}{r}\frac{dw}{dr} \right] r dr d\theta$$

$$- P \times w\big|_{r=0} + \pi \int_A N_r \left(\frac{dw}{dr}\cos\theta \right) r dr d\theta$$

$$w(r) = A(b^2 - r^2)^2 = A(b^4 - 2b^2 r^2 + r^4)$$

$$\frac{dw}{dr} = A(-4b^2 r + 4r^3)$$

$$\frac{d^2 w}{dr^2} = A(-4b^2 + 12r^2)$$

$$w\big|_{r=0} = Ab^4$$

$$\Rightarrow \Pi = \pi D \int_0^{2\pi}\int_0^b \Big\{ \left[A(-4b^2 + 12r^2) + A(-4b^2 + 4r^2) \right]^2$$

$$- 2(1-\nu) \times A(-4b^2 + 12r^2) \times A(-4b^2 + 4r^2) \Big\} dr d\theta$$

$$- P \times Ab^4 + \pi \int_0^{2\pi}\int_0^b A N_r (-4b^2 r + 4r^3) \times r\cos\theta \, dr d\theta$$

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$$\Pi = \pi D \times 64A^2 \int_0^{2\pi}\int_0^b \left[(4r^4 - 4r^2b^2 + b^4) - \frac{1-\nu}{8}(3r^4 - 4r^2b^2 + b^4)\right] dr\,d\theta$$

$$- PAb^4 + \pi A \int_0^{2\pi}\int_0^b N_r (4r^4 - 4b^2r^2) \cos\theta \, dr\,d\theta$$

$$\Pi = \pi D (64A^2) \times 2\pi \left\{\left(\frac{4}{5}r^5 - \frac{4}{3}r^3b^2 + b^4 r\right) - \frac{1-\nu}{8}\left(\frac{3}{5}r^5 - \frac{4}{3}r^3b^2 + b^4 r\right)\right]_0^b$$

$$- PAb^4 + \pi A \left[\frac{4}{5}r^5 - \frac{4}{3}b^2r^3\right]_0^b \times N_r \times \underbrace{\int_0^{2\pi} \cos\theta \, d\theta}_{\simeq 1 \text{ (تقریبا)}}$$

$$\Pi = 128\pi^2 D A^2 \left(\left(\frac{4}{5}b^5 - \frac{4}{3}b^5 + b^5\right) - \frac{1-\nu}{8}\left(\frac{3}{5}b^5 - \frac{4}{3}b^5 + b^5\right)\right)$$

$$- PAb^4 + 2\pi^2 A N_r \left(\frac{4}{5}b^5 - \frac{4}{3}b^5\right)$$

$$\Pi = 128\pi^2 A^2 D \left(\frac{7}{15}b^5 - \frac{1-\nu}{8} \times \frac{4}{15}b^5\right) - PAb^4 - 2\pi^2 A N_r \times \frac{8}{15}b^5$$

$$N_{cr} = -N_r \longrightarrow \Pi = \frac{128}{15}\pi^2 A^2 D b^5 \left(7 - \frac{1-\nu}{8} \times 4\right) - PAb^2 + 2\pi^2 A N_{cr} \frac{8}{15} b^5$$

$$\rightarrow \Pi = \frac{64}{15}\pi^2 A^2 D b^5 (13-\nu) - PAb^2 + \frac{16}{15}\pi^2 A N_{cr} b^5$$

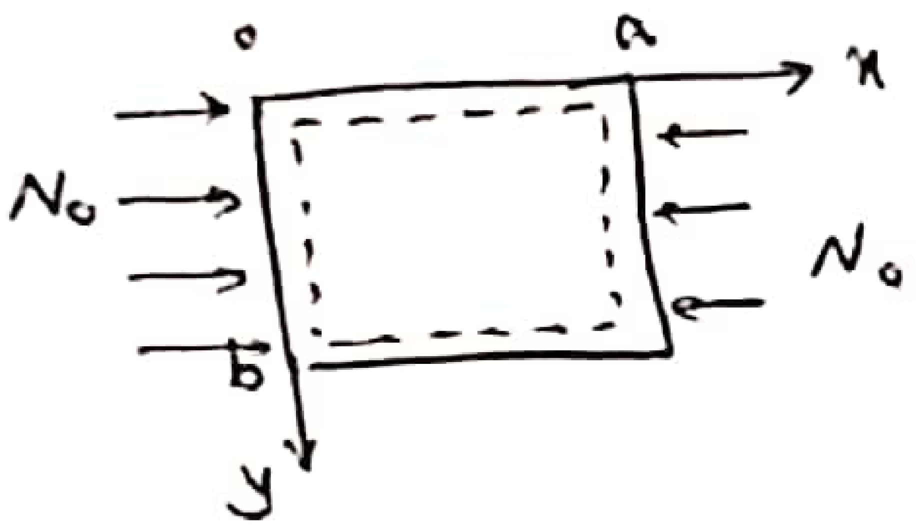
$$\frac{\partial \Pi}{\partial A} = 0 \longrightarrow \frac{128}{15}\pi^2 A D b^5 (13-\nu) - Pb^2 + \frac{16}{15}\pi^2 N_{cr} b^5$$

$$\rightarrow N_{cr} = \frac{Pb^2 - \frac{128}{15}\pi^2 A D b^5 (13-\nu)}{\frac{16}{15}\pi^2 b^5}$$

$$\rightarrow N_{cr} = \frac{15}{16}\frac{P}{\pi^2 b^3} - 8AD(13-\nu)$$

③

(جواب سؤال ۲



$$P(x,y) = P_0 \frac{x}{a}$$

روش نویر : $w(x,y) = \sum\sum W_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$

معادله تعادل: $\nabla^4 w = \frac{1}{D}\left(P + N_x w_{,xx} + \cancel{N_y w_{,yy}} + 2\cancel{N_{xy} w_{,xy}}\right)$

$$P(x,y) = \sum\sum P_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$P_{mn} = \frac{4}{ab}\int_0^a\int_0^b P(x,y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\,dx\,dy$$

$$\rightarrow P_{mn} = \frac{4}{ab}\int_0^a\int_0^b P_0 \frac{x}{a} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\,dx\,dy$$

$$\rightarrow P_{mn} = \frac{4P_0}{a^2 b}\int_0^a\int_0^b x \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\,dx\,dy$$

$$\rightarrow P_{mn} = \frac{4P_0}{a^2 b}\left[\frac{a}{\pi^2 m^2}\left(a\sin\frac{m\pi x}{a} - \pi m x \cos\frac{m\pi x}{a}\right)\right]_0^a \times \left[\frac{-b}{\pi n}\cos\frac{n\pi y}{b}\right]_0^b$$

$$\rightarrow P_{mn} = \frac{4P_0}{a^2 b}\left[\frac{a}{\pi^2 m^2}\left(a\times 0 - \pi m a \times (-1)^m - 0 + 0\right)\right] \times \left[\frac{-b}{\pi n}\left((-1)^n - 1\right)\right]$$

$$\rightarrow P_{mn} = \frac{4P_0}{a^2 \cancel{b}} \times \frac{\cancel{a}}{\pi^2 m^2} \times \left(-\cancel{\pi} m \cancel{a} \times (-1)^m\right) \times \left(\frac{-\cancel{b}}{\cancel{\pi} n}\right)\left((-1)^n - 1\right)$$

$$\rightarrow P_{mn} = \frac{4P_0}{\pi^2 mn} \times (-1)^m \left[(-1)^n - 1\right]$$

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1 * $\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \sum\sum W_{mn}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$

2 * $\frac{P}{D} = \sum\sum \frac{4P_0}{\pi^2 mn D}(-1)^m\left[(-1)^n - 1\right] \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$

3 * $\frac{N_x}{D} w_{,xx} = \sum\sum \frac{-N_0}{D}\left(\frac{m\pi}{a}\right)^2 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{2} \times W_{mn}$

$\boxed{* N_x = -N_0}$

$$\nabla^4 w = \frac{P}{D} + \frac{N_x w_{,xx}}{D} \Rightarrow$$

$$W_{mn}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 = \frac{4P_0}{\pi^2 mn D}(-1)^m\left[(-1)^n - 1\right] + \left(\frac{-N_0}{D}\right)\left(\frac{m\pi}{a}\right)^2 \times W_{mn}$$

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$$W_{mn} = \frac{\frac{4P_0}{\pi^2 mnD}(-1)^m\left[(-1)^n-1\right]}{\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2+\frac{N_0}{D}\left(\frac{m\pi}{a}\right)^2}$$

Buckling : $W_{mn} \to \infty \Rightarrow \left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2 = \frac{-N_{0,cr}}{D}\left(\frac{m\pi}{a}\right)^2$

$$\Rightarrow \quad N_{0,cr} = -D\left(\frac{a}{m\pi}\right)^2 \times \left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2$$

$$\Rightarrow \quad N_{0,cr} = -D\pi^2\left(\frac{a}{m}\right)^2\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2$$