(۱)

تکلیف چهارم از درس تئوری ورق‌ها و پوسته

سؤال ۱)

معادلات تعادل نیرویی

$$
\begin{cases}
\dfrac{\partial}{\partial \alpha}(BN_\alpha)+\dfrac{\partial}{\partial \beta}(AN_{\beta\alpha})+\dfrac{\partial A}{\partial \beta}N_{\alpha\beta}-\dfrac{\partial B}{\partial \alpha}N_\beta-\dfrac{AB}{R_\alpha}Q_\alpha+ABP_\alpha=\rho t AB\ddot{u}_\alpha \\[2ex]
\dfrac{\partial}{\partial \beta}(AN_\beta)+\dfrac{\partial}{\partial \alpha}(BN_{\alpha\beta})+\dfrac{\partial B}{\partial \alpha}N_{\beta\alpha}-\dfrac{\partial A}{\partial \beta}N_\alpha-\dfrac{AB}{R_\beta}Q_\beta+ABP_\beta=\rho t AB\ddot{u}_\beta \\[2ex]
\dfrac{AB}{R_\alpha}N_\alpha+\dfrac{AB}{R_\beta}N_\beta+\dfrac{\partial}{\partial \alpha}(BQ_\alpha)+\dfrac{\partial}{\partial \beta}(AQ_\beta)+ABP_n=\rho t AB\ddot{u}_n
\end{cases}
$$

معادلات تعادل گشتاوری

$$
\begin{cases}
\dfrac{\partial}{\partial \beta}(AM_\beta)+\dfrac{\partial}{\partial \alpha}(BM_{\alpha\beta})+\dfrac{\partial B}{\partial \alpha}M_{\beta\alpha}-\dfrac{\partial A}{\partial \beta}M_\alpha-ABQ_\beta+ABm_\alpha=I\ddot{\alpha} \\[2ex]
\dfrac{\partial}{\partial \alpha}(BM_\alpha)+\dfrac{\partial}{\partial \beta}(AM_{\beta\alpha})+\dfrac{\partial A}{\partial \beta}M_{\alpha\beta}-\dfrac{\partial B}{\partial \alpha}M_\beta-ABQ_\alpha+ABm_\beta=I\ddot{\beta} \\[2ex]
N_{\alpha\beta}-N_{\beta\alpha}-\dfrac{M_{\alpha\beta}}{R_\alpha}+\dfrac{M_{\beta\alpha}}{R_\beta}+m_n=I\ddot{n}
\end{cases}
$$

الف ۱) تنش غشایی

$$M_{\alpha\beta}=M_{\beta\alpha}=Q_\alpha=Q_\beta=0$$

$$
\begin{cases}
\dfrac{\partial}{\partial \alpha}(BN_\alpha)+\dfrac{\partial}{\partial \beta}(AN_{\beta\alpha})+\dfrac{\partial A}{\partial \beta}N_{\alpha\beta}-\dfrac{\partial B}{\partial \alpha}N_\beta+ABP_\alpha=\rho t AB\ddot{u}_\alpha \\[2ex]
\dfrac{\partial}{\partial \beta}(AN_\beta)+\dfrac{\partial}{\partial \alpha}(BN_{\alpha\beta})+\dfrac{\partial B}{\partial \alpha}N_{\beta\alpha}-\dfrac{\partial A}{\partial \beta}N_\alpha+ABP_\beta=\rho t AB\ddot{u}_\beta \\[2ex]
\dfrac{AB}{R_\alpha}N_\alpha+\dfrac{AB}{R_\beta}N_\beta+ABP_n=\rho t AB\ddot{u}_n
\end{cases}
$$

$$
\begin{cases}
ABm_\alpha=I\ddot{\alpha} \\[1ex]
ABm_\beta=I\ddot{\beta} \\[1ex]
N_{\alpha\beta}-N_{\beta\alpha}+m_n=I\ddot{n}
\end{cases}
$$

الف ۲) مسئله استاتیکی

$$\ddot{u}_\alpha=\ddot{u}_\beta=\ddot{u}_n=\ddot{\alpha}=\ddot{\beta}=\ddot{n}=0$$

②

$$
\begin{cases}
\dfrac{\partial}{\partial \alpha}(BN_\alpha) + \dfrac{\partial}{\partial \beta}(AN_{\beta\alpha}) + \dfrac{\partial A}{\partial \beta} N_{\alpha\beta} - \dfrac{\partial B}{\partial \alpha} N_\beta + AB\,P_\alpha = 0 \\[2ex]
\dfrac{\partial}{\partial \beta}(AN_\beta) + \dfrac{\partial}{\partial \alpha}(BN_{\alpha\beta}) + \dfrac{\partial B}{\partial \alpha} N_{\beta\alpha} - \dfrac{\partial A}{\partial \beta} N_\alpha + AB\,P_\beta = 0 \\[2ex]
\dfrac{AB}{R_\alpha} N_\alpha - \dfrac{AB}{R_\beta} N_\beta + AB\,P_n = 0 \longrightarrow \dfrac{N_\alpha}{R_\alpha} - \dfrac{N_\beta}{R_\beta} + P_n = 0
\end{cases}
$$

$$
\begin{cases}
AB\, m_\alpha = 0 \longrightarrow m_\alpha = 0 \\
AB\, m_\beta = 0 \longrightarrow m_\beta = 0 \\
N_{\alpha\beta} - N_{\beta\alpha} + m_n = 0
\end{cases}
$$

الف ۳) تقارن حول β و $P_n = 0$

$P_n = 0$

تقارن $\longrightarrow \dfrac{\partial}{\partial \beta} = 0 \;,\quad N_{\alpha\beta} = N_{\beta\alpha} = P_\beta = m_\beta = 0$

$$
\begin{cases}
\dfrac{\partial}{\partial \alpha}(BN_\alpha) - \dfrac{\partial B}{\partial \alpha} N_\beta + AB\,P_\alpha = 0 \quad \text{(I)} \\[2ex]
0 = 0 \\[1ex]
\dfrac{N_\alpha}{R_\alpha} - \dfrac{N_\beta}{R_\beta} = 0 \longrightarrow \dfrac{N_\alpha}{R_\alpha} = \dfrac{N_\beta}{R_\beta} \longrightarrow N_\alpha = \dfrac{R_\alpha}{R_\beta} N_\beta \quad \text{(II)}
\end{cases}
$$

ب)

$$\text{(II) in (I)} \longrightarrow \frac{\partial}{\partial \alpha}\left(B \frac{R_\alpha}{R_\beta} N_\beta\right) - \frac{\partial B}{\partial \alpha} N_\beta + AB\,P_\alpha = 0$$

$$\longrightarrow \frac{\partial}{\partial \alpha}\left(B \frac{R_\alpha}{R_\beta}\right) N_\beta + B \frac{R_\alpha}{R_\beta} \cdot \frac{\partial}{\partial \alpha}(N_\beta) - \frac{\partial B}{\partial \alpha} N_\beta + AB\,P_\alpha = 0$$

مرتب سازی $\longrightarrow \bar{A} \dfrac{\partial}{\partial \alpha}(N_\beta) + \bar{B} N_\beta + \bar{C} = 0$

$$\bar{A} = B \frac{R_\alpha}{R_\beta} \;,\quad \bar{B} = \frac{\partial}{\partial \alpha}\left(B \frac{R_\alpha}{R_\beta}\right) - \frac{\partial B}{\partial \alpha} \;,\quad \bar{C} = AB\,P_\alpha$$

مرتب سازی $\longrightarrow \dfrac{\partial}{\partial \alpha}(N_\beta) + \dfrac{\bar{A}}{\bar{B}} N_\beta = \dfrac{-\bar{C}}{\bar{A}}$

$$* \quad y' + Py = q \longrightarrow \rho = \exp\left(\int P\,dy\right) \longrightarrow \rho y = \int \rho q\,dy + c'$$

$$\rho = \exp\left(\int \frac{\bar{A}}{\bar{B}}\, d\alpha\right)$$

$$N_\beta = \frac{1}{\rho} \int \frac{\rho(-\bar{C})}{\bar{A}}\, d\alpha + c'$$

③

$$\Rightarrow N_\beta = \frac{-1}{\exp\left(\int \frac{\bar{A}}{\bar{B}} d\alpha\right)} \int \frac{\exp\left(\int \frac{\bar{A}}{\bar{B}} d\alpha\right) \times \bar{C}}{\bar{A}} + c'$$

$$\Rightarrow N_\alpha = \frac{-R_\alpha}{R_\beta \times \exp\left(\int \frac{\bar{A}}{\bar{B}} d\alpha\right)} \int \frac{\exp\left(\int \frac{\bar{A}}{\bar{B}} d\alpha\right) \times \bar{C}}{\bar{A}} + c''$$

$$* \; c'' = \frac{R_\alpha}{R_\beta} c'$$

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$$\begin{cases} \varepsilon_\alpha = \dfrac{1}{1 - z/R_\alpha} (e_\alpha - z k_\alpha) \\ \varepsilon_\beta = \dfrac{1}{1 - z/R_\beta} (e_\beta - z k_\beta) \\ \gamma_{\alpha\beta} = \dfrac{1}{(1 - z/R_\alpha)(1 - z/R_\beta)} \left[\left(1 - \dfrac{z^2}{R_\alpha R_\beta}\right) e_{\alpha\beta} - z \left(1 - \dfrac{z}{2R_\alpha} - \dfrac{z}{2R_\beta}\right) k_{\alpha\beta} \right] \end{cases}$$

کرنش‌های میانی :

$$\begin{cases} e_\alpha = \dfrac{u_{,\alpha}}{A} + \dfrac{v}{AB} A_{,\beta} - \dfrac{w}{R_\alpha} \\ e_\beta = \dfrac{u}{AB} B_{,\alpha} + \dfrac{1}{B} v_{,\beta} - \dfrac{w}{R_\beta} \\ e_{\alpha\beta} = \dfrac{A}{B} \dfrac{\partial}{\partial \beta} \left(\dfrac{u}{A}\right) + \dfrac{B}{A} \dfrac{\partial}{\partial \alpha} \left(\dfrac{v}{B}\right) \end{cases}$$

شعاع انحنا :

$$\begin{cases} k_\alpha = \dfrac{1}{A} \theta_{\alpha,\alpha} + \dfrac{\theta_\beta}{AB} A_{,\beta} \\ k_\beta = \dfrac{\theta_\alpha}{AB} B_{,\alpha} + \dfrac{1}{B} \theta_{\beta,\beta} \\ k_{\alpha\beta} = \dfrac{A}{B} \dfrac{\partial}{\partial \beta} \left(\dfrac{\theta_\alpha}{A}\right) + \dfrac{B}{A} \dfrac{\partial}{\partial \alpha} \left(\dfrac{\theta_\beta}{B}\right) + \dfrac{1}{R_\alpha} \left(\dfrac{u_{,\beta}}{B} - \dfrac{v}{AB} B_{,\alpha}\right) \\ \qquad + \dfrac{1}{R_\beta} \left(\dfrac{v_{,\alpha}}{A} - \dfrac{u}{AB} A_{,\beta}\right) \end{cases}$$

$$* \; \theta_\alpha = \frac{u}{R_\alpha} + \frac{w_{,\alpha}}{A}$$

$$* \; \theta_\beta = \frac{v}{R_\beta} + \frac{w_{,\beta}}{B}$$

$$\frac{z}{R_\alpha}, \frac{z}{R_\beta} \ll 1$$

$$\begin{cases} \varepsilon_\alpha = e_\alpha - z k_\alpha \\ \varepsilon_\beta = e_\beta - z k_\beta \\ \gamma_{\alpha\beta} = e_{\alpha\beta} - z k_{\alpha\beta} \end{cases}$$

کرنش لایه میانی و شعاع انحنا همان موارد ذکر شده صفحه قبل است!

ج. ۲) تقارن حول β

$$\frac{\partial}{\partial\beta} = 0 \quad , \quad v = 0 \quad , \quad \theta_\beta = 0$$

$$\begin{cases} \varepsilon_\alpha = e_\alpha - z k_\alpha \\ \varepsilon_\beta = e_\beta - z k_\beta \\ \gamma_{\alpha\beta} = e_{\alpha\beta} - z k_{\alpha\beta} \end{cases}$$

کرنش لایه میانی:

$$\begin{cases} e_\alpha = \dfrac{u_{,\alpha}}{A} - \dfrac{w}{R_\alpha} \\ e_\beta = \dfrac{u}{AB} B_{,\alpha} - \dfrac{w}{R_\beta} \\ e_{\alpha\beta} = 0 \end{cases}$$

شعاع انحنا:

$$\begin{cases} k_\alpha = \dfrac{1}{A}\dfrac{\partial}{\partial\alpha}\left(\dfrac{u}{R_\alpha} + \dfrac{w_{,\alpha}}{A}\right) \\ k_\beta = \dfrac{1}{AB}\left(\dfrac{u}{R_\alpha} + \dfrac{w_{,\alpha}}{A}\right) B_{,\alpha} \\ k_{\alpha\beta} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \varepsilon_\alpha = e_\alpha - z k_\alpha = \dfrac{u_{,\alpha}}{A} - \dfrac{w}{R_\alpha} - \dfrac{z}{A}\dfrac{\partial}{\partial\alpha}\left(\dfrac{u}{R_\alpha} + \dfrac{w_{,\alpha}}{A}\right) \\ \varepsilon_\beta = e_\beta - z k_\beta = \dfrac{u}{AB} B_{,\alpha} - \dfrac{w}{R_\beta} - \dfrac{z}{AB}\left(\dfrac{u}{R_\alpha} + \dfrac{w_{,\alpha}}{A}\right) B_{,\alpha} \\ \gamma_{\alpha\beta} = 0 \end{cases}$$

با اعمال شرایط الف تا ج

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$$\sigma_\alpha = \frac{N_\alpha}{t} + \frac{12 M_\alpha}{t^3} z = \frac{N_\alpha}{t}$$

$$\sigma_\beta = \frac{N_\beta}{t} + \frac{12 M_\beta}{t^3} z = \frac{N_\beta}{t}$$

$$\tau_{\alpha\beta} = \frac{N_{\alpha\beta}}{t} + \frac{12 M_{\alpha\beta}}{t^3} z = 0$$

$$\tau_{\alpha z} = \frac{3 Q_\alpha}{2t}\left(1 - \frac{4z^2}{t^2}\right) = 0$$

$$\tau_{\beta z} = \frac{3 Q_\beta}{2t}\left(1 - \frac{4z^2}{t^2}\right) = 0$$

$$\sigma_\alpha = \frac{E}{1-\nu^2}(\varepsilon_\alpha + \nu \varepsilon_\beta) = \frac{N_\alpha}{t}$$

$$\sigma_\beta = \frac{E}{1-\nu^2}(\varepsilon_\beta + \nu \varepsilon_\alpha) = \frac{N_\beta}{t}$$

$$\tau_{\alpha\beta} = \frac{E}{2(1+\nu)} \gamma_{\alpha\beta} = 0$$

$$\text{I} \begin{cases} N_\alpha = \tilde{C}(e_\alpha + \nu e_\beta) \\ N_\beta = \tilde{C}(e_\beta + \nu e_\alpha) \end{cases} \qquad * \ \tilde{C} = \frac{Et}{1-\nu^2}$$

$$\text{II} \begin{cases} M_\alpha = -\tilde{D}(k_\alpha + \nu k_\beta) = 0 \\ M_\beta = -\tilde{D}(k_\beta + \nu k_\alpha) = 0 \end{cases} \qquad * \ \tilde{D} = \frac{Et^3}{12(1-\nu^2)}$$

$$\text{I} \rightarrow \begin{cases} e_\alpha + \nu e_\beta = \dfrac{N_\alpha}{\tilde{C}} \\ e_\beta + \nu e_\alpha = \dfrac{N_\beta}{\tilde{C}} \end{cases}$$

$$\text{II} \rightarrow \begin{cases} k_\alpha + \nu k_\beta = 0 \\ k_\beta + \nu k_\alpha = 0 \end{cases}$$

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$$\mathrm{I} \longrightarrow \begin{cases} -\nu e_\alpha - \nu^2 e_\beta = \dfrac{-\nu}{\tilde{C}} N_\alpha \\ e_\beta + \nu e_\alpha = \dfrac{1}{\tilde{C}} N_\beta \end{cases}$$

$$(1-\nu^2)\, e_\beta = \frac{1}{\tilde{C}} (N_\beta - \nu N_\alpha)$$

$$\Rightarrow \quad e_\beta = \frac{N_\beta - \nu N_\alpha}{\tilde{C}(1-\nu^2)} = \frac{N_\beta - \nu N_\alpha}{Et}$$

همین ترتیب $\Rightarrow \quad e_\alpha = \dfrac{N_\alpha - \nu N_\beta}{Et}$

$$\mathrm{II} \longrightarrow \begin{cases} k_\alpha = 0 \\ k_\beta = 0 \end{cases}$$

از قبل $\begin{cases} e_{\alpha\beta} = 0 \\ k_{\alpha\beta} = 0 \end{cases}$

$$\begin{cases} e_\alpha = \dfrac{N_\alpha - \nu N_\beta}{Et} \\ e_\beta = \dfrac{N_\beta - \nu N_\alpha}{Et} \\ e_{\alpha\beta} = 0 \end{cases}$$

$$\begin{cases} k_\alpha = 0 \\ k_\beta = 0 \\ k_{\alpha\beta} = 0 \end{cases}$$

سؤال ۲)

از مسئله قبل بدیهیست که $(P_n \neq 0)$

$$\begin{cases} \dfrac{\partial}{\partial \alpha}(B N_\alpha) - \dfrac{\partial B}{\partial \alpha} N_\beta + A B P_\alpha = 0 \\[2mm] \dfrac{N_\alpha}{R_\alpha} - \dfrac{N_\beta}{R_\beta} + P_n = 0 \end{cases}$$

بجای معادله اول می‌توان از رابطه زیر استفاده نمود:

$$N_\alpha = \frac{-F}{2\pi r_0 \sin\varphi} \quad , \quad P_\beta = 0 \ (\text{تقارن}) \quad , \quad P_\alpha \simeq 0$$

مخزن کروی

$$\begin{cases} \alpha \equiv \varphi \\ \beta \equiv \theta \end{cases} \qquad \begin{cases} A = a \\ B = r_0 = a\sin\varphi \end{cases}$$

$$\begin{cases} R_\alpha = r_1 = a \\ R_\beta = r_2 = a \end{cases}$$

$$\rightarrow \begin{cases} \dfrac{N_\varphi}{a} + \dfrac{N_\theta}{a} + P_z = 0 \\[2mm] N_\varphi = \dfrac{-F}{2\pi (a\sin\varphi)\sin\varphi} \end{cases}$$

$$P_z = -P_0$$

$$F = -\int_0^\varphi dF = -\int_0^\varphi P_0 (2\pi r_0 \, a\, d\varphi)$$

$$\downarrow \ \sin\varphi\cos\theta$$

$$\rightarrow F = -\int_0^\varphi P_0 2\pi a^2 \sin^2\varphi \cos\theta \, d\varphi$$

$$\rightarrow F = -P_0 2\pi a^2 \cos\theta \int_0^\varphi \sin^2\varphi \, d\varphi$$

$$\rightarrow F = -P_0 2\pi a^2 \cos\theta \left[\frac{\varphi}{2} - \frac{\sin 2\varphi}{2}\right]_0^\varphi$$

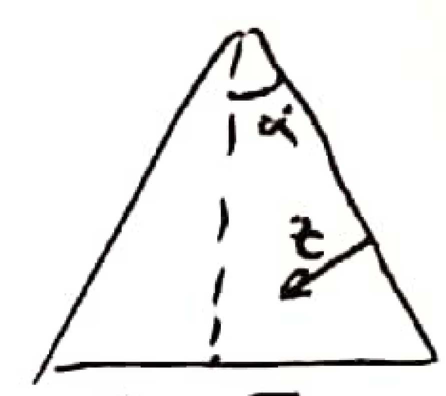
$$\rightarrow F = -P_0 2\pi a^2 \cos\theta \left[\frac{\varphi}{2} - \frac{\sin 2\varphi}{2}\right]$$

$$\rightarrow F = -P_0 \pi a^2 \cos\theta (\varphi - \sin 2\varphi)$$

مخزن مخروطی

$$\begin{cases} \alpha \equiv x \\ \beta \equiv \theta \end{cases} \qquad \begin{cases} A = 1 \\ B = r_0 = x\sin\alpha \end{cases}$$

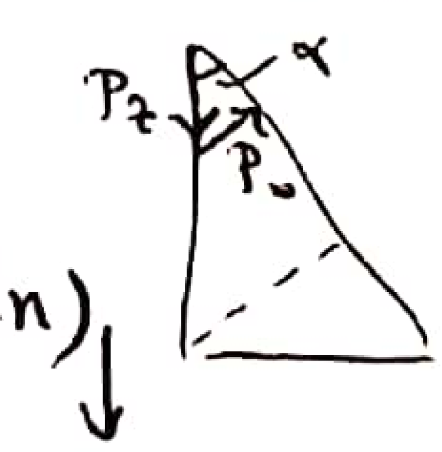
$$\begin{cases} R_\alpha = R_1 = \infty \\ R_\beta = r_2 = x\tan\alpha \end{cases}$$



$$\varphi = \frac{\pi}{2} - \alpha$$

$$\rightarrow \begin{cases} \dfrac{N_x}{\infty} + \dfrac{N_\theta}{x\tan\alpha} + P_z = 0 \\[2mm] N_x = \dfrac{-F}{2\pi (x\sin\alpha)\sin(\frac{\pi}{2} - \alpha)} \end{cases}$$

$$P_z = -P_0 \frac{1}{\sin\alpha}$$



$$F = -\int_0^\varphi dF = -\int_0^\varphi P_0 (2\pi r_0 \, dx)$$

$$\downarrow \ \sin\varphi\cos\theta$$

$$\rightarrow F = -\int_0^{\pi/2-\alpha} P_0 2\pi x \sin\alpha \sin\left(\frac{\pi}{2} - \alpha\right) \cos\theta \, dx$$

$$\rightarrow F = -2\pi \cos\theta P_0 \int_0^{\pi/2-\alpha} x \sin\alpha \cos\alpha \, dx$$

$$\rightarrow F = -P_0 2\pi \cos\theta \sin\alpha \cos\alpha \left[\frac{x^2}{2}\right]_0^{\pi/2-\alpha}$$

$$\rightarrow F = -P_0 2\pi \cos\theta \times \frac{1}{2}\sin 2\alpha \left[\frac{1}{2}\left(\frac{\pi}{2} - \alpha\right)^2\right]$$

$$\rightarrow F = \frac{-P_0}{2} \pi \cos\theta \sin 2\alpha \left(\frac{\pi}{2} - \alpha\right)^2$$

⑧

کره

$$N_\varphi = \frac{P_0 \pi a^2 \cos\theta \,(\varphi - \sin 2\varphi)}{2\pi (a\sin\varphi)\sin\varphi}$$

$$\rightarrow N_\varphi = \frac{P_0}{2} a\cos\theta \times \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}$$

مخروط

$$N_x = \frac{\frac{P_0}{2}\pi \cos\theta \sin 2\alpha \,(\pi/2 - \alpha)^2}{2\pi (x\sin\alpha)\cos\alpha}$$

$$\rightarrow N_x = \frac{P_0}{4}\cos\theta \times \frac{\sin 2\alpha\,(\pi/2-\alpha)^2}{x \sin\alpha\cos\alpha}$$

$$\rightarrow N_x = \frac{P_0}{2}\cos\theta \times \frac{(\pi/2-\alpha)^2}{x}$$

$$\rightarrow \frac{N_\varphi}{a} + \frac{N_\theta}{a} + P_z = 0$$

$$\rightarrow N_\theta = -a\left(-P_0 - \frac{P_0}{2} a\cos\theta \times \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}\right)$$

$$\rightarrow N_\theta = aP_0\left(1 + \frac{1}{2} a\cos\theta\, \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}\right)$$

$$\rightarrow \frac{N_x}{\infty} + \frac{N_\theta}{x\tan\alpha} + P_z = 0$$

$$\rightarrow N_\theta = -x\tan\alpha\left(-P_0 \frac{1}{\sin\alpha}\right)$$

$$\rightarrow N_\theta = \frac{P_0 x}{\cos\alpha}$$

از مسئله قبل:

$$\sigma_\alpha = \frac{N_\alpha}{t} \qquad e_\alpha = \frac{N_\alpha - \nu N_\beta}{Et}$$

$$\sigma_\beta = \frac{N_\beta}{t} \qquad e_\beta = \frac{N_\beta - \nu N_\alpha}{Et}$$

$$\tau_{\alpha\beta} = 0 \qquad e_{\alpha\beta} = 0$$

$$k_\alpha = k_\beta = k_{\alpha\beta} = 0$$

کره

$$\sigma_\varphi = \frac{N_\varphi}{t} = \frac{1.5\times10^6}{2\times0.045} \times 0.5\cos\theta\, \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}$$

$$\rightarrow \sigma_\varphi = 8.3\times10^6 \cos\theta\, \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}$$

$$\sigma_\theta = \frac{0.5}{0.045}\times 1.5\times10^6\left(1 + \frac{1}{2}\times 0.5\cos\theta\, \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}\right)$$

$$\rightarrow \sigma_\theta = 16.7\times10^6\left(1 + \frac{1}{4}\cos\theta\, \frac{\varphi - \sin 2\varphi}{\sin^2\varphi}\right)$$

مخروط

$$\sigma_x = \frac{1.5\times10^6}{2\times0.0032}\cos\theta \times \frac{(\pi/2-\alpha)^2}{x}$$

$$\rightarrow \sigma_x = 234.4\times10^6 \cos\theta\, \frac{(\pi/2-\alpha)^2}{x}$$

$$\sigma_\theta = \frac{1.5\times10^6 \times x}{0.0032\times\cos\alpha}$$

$$\rightarrow \sigma_\theta = 468.8\times10^6\, \frac{x}{\cos\alpha}$$

$$\alpha = 37^\circ \rightarrow \begin{cases} \sigma_x = 200.4\times10^6\, \dfrac{\cos\theta}{x} \\[2ex] \sigma_\theta = 587.0\times10^6\, x \end{cases}$$

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کره

$$e_\varphi = \frac{1}{200\times10^9}\left[8.3\times10^6 \cos\theta \,\frac{\varphi-\sin2\varphi}{\sin^2\varphi} - 0.27\times16.7\times10^6\left(1+\frac{1}{4}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}\right)\right]$$

$$\rightarrow e_\varphi = 41.5\times10^{-6}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi} - 22.5\times10^{-6}\left(1+\frac{1}{4}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}\right)$$

$$e_\theta = \frac{1}{200\times10^9}\left[16.7\times10^6\left(1+\frac{1}{4}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}\right) - 0.27\times8.3\times10^6\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}\right]$$

$$\rightarrow e_\theta = 83.5\times10^{-6}\left(1+\frac{1}{4}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}\right) - 11.2\times10^{-6}\cos\theta\,\frac{\varphi-\sin2\varphi}{\sin^2\varphi}$$

مخروط

$$e_x = \frac{1}{200\times10^9}\left[200.4\times10^6\,\frac{\cos\theta}{x} - 0.27\times587.0\times10^6\,x\right]$$

$$\rightarrow e_x = 1002.0\times10^{-6}\,\frac{\cos\theta}{x} - 792.5\times10^{-6}\,x$$

$$e_\theta = \frac{1}{200\times10^9}\left[578.0\times10^6\,x - 0.27\times200.4\times10^6\,\frac{\cos\theta}{x}\right]$$

$$\rightarrow e_\theta = 2890.0\times10^{-6}\,x - 270.5\times10^{-6}\,\frac{\cos\theta}{x}$$

کره

$$\begin{cases}\theta = 0\\ \varphi = \pi/2\end{cases}$$

$$\sigma_\varphi = 8.3\times10^6\times1\times\frac{\pi/2-0}{1} = 13.0 \text{ MPa}$$

$$\sigma_\theta = 16.7\times10^6\left(1+\frac{1}{4}\times1\times\frac{\pi/2-0}{1}\right) = 23.3 \text{ MPa}$$

مخروط

$$\begin{cases}x = \dfrac{0.55}{\sin\alpha} = 0.91 \text{ m}\\ \varphi = \dfrac{\pi}{2}-\alpha = 53^\circ\end{cases}$$

$$\sigma_x = 200.4\times10^6\times\frac{1}{0.91} = 220.2 \text{ MPa}$$

$$\sigma_\theta = 587.0\times10^6\times0.91 = 534.2 \text{ MPa}$$

تنش ها در مخزن مخروطی بیشتر از مخزن کروی است!

لذا زودتر دچار واماندگی می شود.