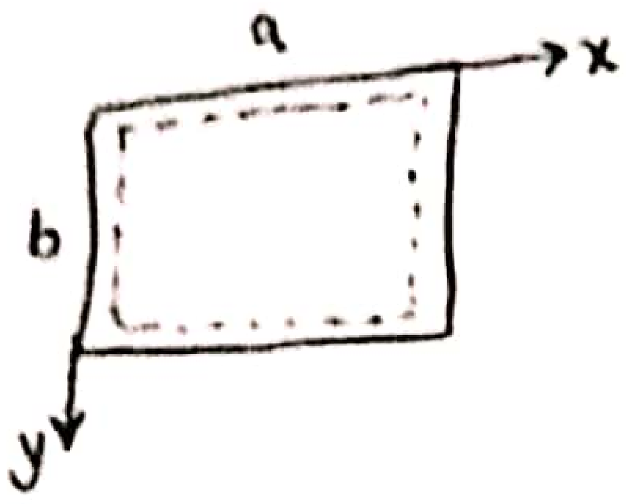
تئوری ورق و پوسته

Take-Home # 1



جواب سئوال 1) الف

$\boxed{1}$ $x=0 \rightarrow w=0\ ,\ M_x=0 \rightarrow w_{,xx}=0$

$x=0 \rightarrow w = A(0)(\cdots) = 0$ OK ✓

$$w_{,x} = A\left(4\frac{x^3}{a^4} - 6\frac{x^2}{a^3} + 2\frac{x}{a^2}\right)\left(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\right)$$

$$w_{,xx} = A\left(12\frac{x^2}{a^4} - 12\frac{x}{a^3} + \frac{2}{a^2}\right)\left(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\right)$$

$x=0 \rightarrow w_{,xx} = A\left(0 - 0 + \frac{2}{a^2}\right)(\cdots) \neq 0$ NOK X

$\boxed{2}$ $x=a \rightarrow w=0\ ,\ M_x=0 \rightarrow w_{,xx}=0$

$x=a \rightarrow w = A(1-2+1)(\cdots) = 0$ OK ✓

$x=a \rightarrow w_{,xx} = A\left(\frac{12}{a^2} - \frac{12}{a^2} + \frac{2}{a^2}\right)(\cdots) \neq 0$ NOK X

$\boxed{3}$ $y=0 \rightarrow w=0\ ,\ M_y=0 \rightarrow w_{,yy}=0$

چون جملات پرانتز مربوط به y دقیقاً مانند پرانتز مربوط به x است، پس نتایج مشابه به دست می‌آید.

$y=0 \rightarrow w=0$ OK ✓

$y=0 \rightarrow w_{,yy} \neq 0$ NOK X

$y=b \rightarrow w=0$ OK ✓

$y=b \rightarrow w_{,yy} \neq 0$ NOK X

ب

مکان ماکزیمم خیز ورق $\rightarrow$ $w_{,x} = w_{,y} = 0$

با توجه به تقارن $\rightarrow$ $x = \frac{a}{2}\ ,\ y = \frac{b}{2}$

$$\Rightarrow w_{max} = A\left(\frac{a^4}{16a^4} - 2\frac{a^3}{8a^3} + \frac{a^2}{4a^2}\right)\left(\frac{b^4}{16b^4} - 2\frac{b^3}{8b^3} + \frac{b^2}{4b^2}\right)$$

$$\Rightarrow w_{max} = A\left(\frac{1}{16} - \frac{1}{4} + \frac{1}{4}\right)\left(\frac{1}{16} - \frac{1}{4} + \frac{1}{4}\right)$$

$$\Rightarrow w_{max} = \frac{A}{256}$$

$$M_x = -D(w_{,xx} + \nu w_{,yy})$$

$$M_y = -D(w_{,yy} + \nu w_{,xx})$$

$$M_{xy} = -D(1-\nu)\, w_{,xy}$$

$$M_x = -D\Big[A\Big(12\frac{x^2}{a^4} - 12\frac{x}{a^3} + 2\frac{1}{a^2}\Big)\Big(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\Big) + \nu A\Big(\frac{x^4}{a^4} - 2\frac{x^3}{a^3} + \frac{x^2}{a^2}\Big)\Big(12\frac{y^2}{b^4} - 12\frac{y}{b^3} + 2\frac{1}{b^2}\Big)\Big]$$

$$M_y = -D\Big[A\Big(\frac{x^4}{a^4} - 2\frac{x^3}{a^3} + \frac{x^2}{a^2}\Big)\Big(12\frac{y^2}{b^4} - 12\frac{y}{b^3} + 2\frac{1}{b^2}\Big) + \nu A\Big(12\frac{x^2}{a^4} - 12\frac{x}{a^3} + 2\frac{1}{a^2}\Big)\Big(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\Big)\Big]$$

$$M_{xy} = -D(1-\nu)A\Big[\Big(4\frac{x^3}{a^4} - 6\frac{x^2}{a^3} + 2\frac{x}{a^2}\Big)\Big(4\frac{y^3}{b^4} - 6\frac{y^2}{b^3} + 2\frac{y}{b^2}\Big)\Big]$$

$$\sigma_x = \frac{12 M_x \cdot z}{t^3}, \quad \sigma_y = \frac{12 M_y \cdot z}{t^3}, \quad \tau_{xy} = \frac{12 M_{xy} \cdot z}{t^3}$$

$$\sigma_x = \frac{-12DAz}{t^3}\Big[\Big(12\frac{x^2}{a^4} - 12\frac{x}{a^3} + 2\frac{1}{a^2}\Big)\Big(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\Big) + \nu\Big(\frac{x^4}{a^4} - 2\frac{x^3}{a^3} + \frac{x^2}{a^2}\Big)\Big(12\frac{y^2}{b^4} - 12\frac{y}{b^3} + 2\frac{1}{b^2}\Big)\Big]$$

$$\sigma_y = \frac{-12DAz}{t^3}\Big[\Big(\frac{x^4}{a^4} - 2\frac{x^3}{a^3} + \frac{x^2}{a^2}\Big)\Big(12\frac{y^2}{b^4} - 12\frac{y}{b^3} + 2\frac{1}{b^2}\Big) + \nu\Big(12\frac{x^2}{a^4} - 12\frac{x}{a^3} + 2\frac{1}{a^2}\Big)\Big(\frac{y^4}{b^4} - 2\frac{y^3}{b^3} + \frac{y^2}{b^2}\Big)\Big]$$

$$\tau_{xy} = \frac{-12D(1-\nu)Az}{t^3}\Big[\Big(4\frac{x^3}{a^4} - 6\frac{x^2}{a^3} + 2\frac{x}{a^2}\Big)\Big(4\frac{y^3}{b^4} - 6\frac{y^2}{b^3} + 2\frac{y}{b^2}\Big)\Big]$$

$$a = b$$

$$x = y = \frac{a}{2}$$

$$\sigma_x = \frac{-12DAz}{t^3}\left[\left(\frac{12}{4a^2} - \frac{12}{2a^2} + \frac{2}{a^2}\right)\left(\frac{1}{16} - 2\frac{\cancel{1}}{8} + \frac{\cancel{1}}{4}\right) + \nu\left(\frac{1}{16} - 2\frac{\cancel{1}}{8} + \frac{\cancel{1}}{4}\right)\left(\frac{12}{4a^2} - \frac{12}{2a^2} + \frac{2}{a^2}\right)\right]$$

$$\sigma_x = \frac{-12DAz}{t^3}\left[\left(\frac{-1}{a^2}\right)\left(\frac{1}{16}\right) + \nu\left(\frac{1}{16}\right)\left(\frac{-1}{a^2}\right)\right]$$

$$\sigma_x = \frac{+12DAz}{16a^2t^3}(1+\nu) = \frac{3DAz(1+\nu)}{4a^2t^3}$$

چون معادلات مشابه هستند همین طریق:

$$\sigma_y = \frac{3DAz(1+\nu)}{4a^2t^3}$$

$$\tau_{xy} = \frac{-12D(1-\nu)Az}{t^3}\left[\left(4\frac{1}{8a} - 6\frac{1}{4a} + \frac{2}{2a}\right)\left(4\frac{1}{8a} - 6\frac{1}{4a} + \frac{1}{a}\right)\right]$$

$$\tau_{xy} = \frac{-12D(1-\nu)Az}{t^3}(0) = 0$$

مقادیر تنش در لایه پائینی ورق عبارتند از:

$$z = \frac{+t}{2}$$

$$\begin{cases} \sigma_x = \dfrac{3DA(1+\nu)}{8a^2t^4} \\[2ex] \sigma_y = \dfrac{3DA(1+\nu)}{8a^2t^4} \\[2ex] \tau_{xy} = 0 \end{cases}$$

$$D = \frac{Et^3}{12(1-\nu^2)} \rightarrow \begin{cases} \sigma_x = \dfrac{EA}{32a^2t(1-\nu)} \\[2ex] \sigma_y = \dfrac{EA}{32a^2t(1-\nu)} \\[2ex] \tau_{xy} = 0 \end{cases}$$

طرح ۴

جواب سؤال ۲)

الف

در حالت متقارن : $D\nabla^4 w = \frac{P}{D} \Rightarrow \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{P_0}{D}\cdot\frac{r^2}{a^2}$

$$\Rightarrow \frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{P_0}{a^2 D} r^3$$

$$\Rightarrow r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{P_0}{4a^2 D} r^4 + c_1$$

$$\Rightarrow \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{P_0}{4a^2 D} r^3 + \frac{c_1}{r}$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{P_0}{16a^2 D} r^4 + c_1 \ln r + c_2$$

$$\Rightarrow \frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{P_0}{16a^2 D} r^5 + c_1 r\ln r + c_2 r$$

$$\Rightarrow r\frac{dw}{dr} = \frac{P_0}{96a^2 D} r^6 + c_1\left(\frac{1}{2}r^2\ln r - \frac{r^2}{4}\right) + \frac{c_2}{2}r^2 + c_3$$

$$\Rightarrow \frac{dw}{dr} = \frac{P_0}{96a^2 D} r^5 + c_1\left(\frac{1}{2}r\ln r - \frac{r}{4}\right) + \frac{c_2}{2}r + c_3/r$$

$$\Rightarrow w = \frac{P_0}{576a^2 D} r^6 + c_1\left[\frac{1}{2}\left(\frac{1}{2}r^2\ln r - \frac{r^2}{4}\right) - \frac{r^2}{8}\right] + \frac{c_2}{4}r^2 + c_3\ln r + c_4$$

شرایط مرزی : $r = 0 \longrightarrow w, Q_r \rightarrow$ معین

$r = a \longrightarrow w = w_{,r} = 0$

یا $w = \frac{P_0}{576a^2 D} r^6 + \frac{c_1}{4} r^2 \ln r - c_1\frac{r^2}{4} + c_2\frac{r^2}{4} + c_3\ln r + c_4$

از شرایط مرزی $\longrightarrow$ $\ln r$ ضرایب صفر هستند $\Rightarrow c_1 = c_3 = 0$ ✓

$$r = a \longrightarrow \frac{dw}{dr} = 0 \Rightarrow \frac{P_0}{96a^2 D}(a)^5 + c_2\frac{a}{2} + 0 = 0$$

$$\Rightarrow c_2 = -\frac{P_0 a^2}{48 D} \checkmark$$

$$r = a \longrightarrow w = 0 \Rightarrow \frac{P_0}{576a^2 D}a^6 - \frac{P_0 a^2}{48 D}\times\frac{a^2}{4} + c_4 = 0$$

$$\Rightarrow c_4 = \frac{P_0 a^4}{288 D} \checkmark$$

حل

$$\Rightarrow \quad w = \frac{P_0}{576\,a^2 D} r^6 - \frac{P_0 a^2}{48D} \times \frac{r^2}{4} + \frac{P_0 a^4}{288\,D}$$

$$\Rightarrow \quad w = \frac{P_0 a^4}{576\,D}\left[\left(\frac{r}{a}\right)^6 - 3\left(\frac{r}{a}\right)^2 + 2\right] \qquad \text{OK} \ \checkmark$$

ب

درحالت متقارن : $M_r = -D\left[\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right]$

$$\frac{dw}{dr} = \frac{P_0 a^4}{576\,D}\left[6\frac{r^5}{a^6} - 6\frac{r}{a^2}\right]$$

$$\frac{d^2w}{dr^2} = \frac{P_0 a^4}{576\,D}\left[30\frac{r^4}{a^6} - \frac{6}{a^2}\right]$$

$$\Rightarrow M_r = -D\left[\frac{P_0 a^4}{576\,D}\left(\frac{30r^4}{a^6} - \frac{6}{a^2}\right) + \frac{\nu}{r} \times \frac{P_0 a^4}{576\,D}\left(\frac{6r^5}{a^6} - \frac{6r}{a^2}\right)\right]$$

$$\Rightarrow M_r = -\frac{P_0 a^2}{576}\left[\left(\frac{30\,r^4}{a^4} - 6\right) + \nu\left(\frac{6r^4}{a^4} - 6\right)\right]$$

M_r ماکزیمم $\longrightarrow$ $r = a$ $\Rightarrow$ $(M_r)_{max} = -\frac{P_0 a^2}{576}\left[(30-6) + \nu(\cancel{6-6})\right]$

در لبه‌ی گیردار

$$\Rightarrow (M_r)_{max} = -\frac{3P_0 a^2}{72} \quad \checkmark$$