طراحی اجزای یک
تکیه جسم یک

جواب سوال ۱)

Table A-5

Aluminum $E = 71.7$ GPa , $\nu = 0.333$

Carbon steel $E = 207.0$ GPa , $\nu = 0.292$

(I) تماس کره با کره $a = \sqrt[3]{\frac{3F}{8} \times \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$, $P_{max} = \frac{3F}{2\pi a^2}$

$$a = \sqrt[3]{\frac{3\times 2\times 10^3}{8} \times \frac{(1-0.333^2)/71.7\times 10^9 + (1-0.292^2)/207.0\times 10^9}{1/(150\times 10^{-3}) + 1/(150\times 10^{-3})}} = 0.000982 \text{ m}$$

$$P_{max} = \frac{3\times 2\times 10^3}{2\pi (0.000982)^2} = 990.9 \text{ MPa}$$

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -P_{max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2(1+\frac{z^2}{a^2})}\right]$$

$$\sigma_3 = \sigma_z = \frac{-P_{max}}{1+\frac{z^2}{a^2}}$$

$z = 0 \longrightarrow$ تنش‌های اصلی $\longrightarrow$ $\begin{cases} \sigma_1 = \sigma_2 = -P_{max}\left[(1+\nu) - 1/2\right] \\ \sigma_3 = \frac{-P_{max}}{1+0} = -P_{max} \end{cases}$

$\sigma_1 = -990.9\left[0.5+\nu\right]$ $\begin{cases} -825.4 \text{ MPa} & \text{Aluminum} \\ -784.8 \text{ MPa} & \text{Steel} \end{cases}$

$\sigma_3 = -990.9$ for both Aluminum / steel

$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right|$ $\begin{cases} 82.7 \text{ MPa} & \text{Aluminum} \\ 103.1 \text{ MPa} & \text{Steel} \end{cases}$

(II) تماس کره با صفحه $\begin{cases} \text{کره آلومینیوم و صفحه فولادی} \\ \text{کره فولادی و صفحه آلومینیوم} \end{cases} \longrightarrow d_2 = \infty$

$a = 0.001237$ m

$P_{max} = 624.2$ MPa

$\sigma_1 = -624.2\left[0.5+\nu\right]$ $\begin{cases} -520.0 \text{ MPa} & \text{Aluminum} \\ -494.4 \text{ MPa} & \text{steel} \end{cases}$

$\sigma_3 = -624.2$ MPa for both Aluminum / steel

$\tau_{max} = \begin{cases} 52.1 \text{ MPa} & \text{Aluminum} \\ 64.9 \text{ MPa} & \text{Steel} \end{cases}$

حل

(III) تماس استوانه با استوانه

$$b = \sqrt{\frac{2F}{\pi l} \times \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}} \quad , \quad P_{max} = \frac{2F}{\pi b l}$$

$$b = \sqrt{\frac{2 \times 2 \times 10^3}{\pi \times 150 \times 10^{-3}} \times \frac{(1-0.333^2)/71.7\times10^9 + (1-0.292^2)/207.0\times10^9}{1/150\times10^{-3} + 1/150\times10^{-3}}} = 0.000103\ m$$

$$P_{max} = \frac{2 \times 2 \times 10^3}{\pi \times 0.000103 \times 150 \times 10^{-3}} = 82.0 \ MPa$$

$$0 \leq z \leq 0.436 b \rightarrow \sigma_1 = \sigma_x = -2\nu P_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$

$z = 0 \rightarrow$ ماکزیمم تنش ها $\rightarrow \sigma_1 = -2\nu P_{max}$

$$\sigma_3 = \sigma_z = \frac{-P_{max}}{\sqrt{1+z^2/b^2}}$$

$z = 0 \rightarrow$ ماکزیمم تنش ها $\rightarrow \sigma_3 = -P_{max}$

$\sigma_1 = -2\nu \times 82.0$ <
- -54.6 MPa Aluminum
- -47.9 MPa Steel

$\sigma_3 = -82.0$ MPa for both Aluminum/Steel

$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right|$ <
- 13.7 MPa Aluminum
- 17.1 MPa Steel

(IV) تماس استوانه با صفحه <
- استوانه آلومینیومی و صفحه فولادی
- استوانه فولادی و صفحه آلومینیومی

$\rightarrow d_2 = \infty$

$b = 0.000146$ m

$P_{max} = 58.0$ MPa

$\sigma_1 = -2\nu \times 58.0$ <
- -38.6 MPa Aluminum
- -33.9 MPa Steel

$\sigma_3 = -58.0$ MPa for both Aluminum/Steel

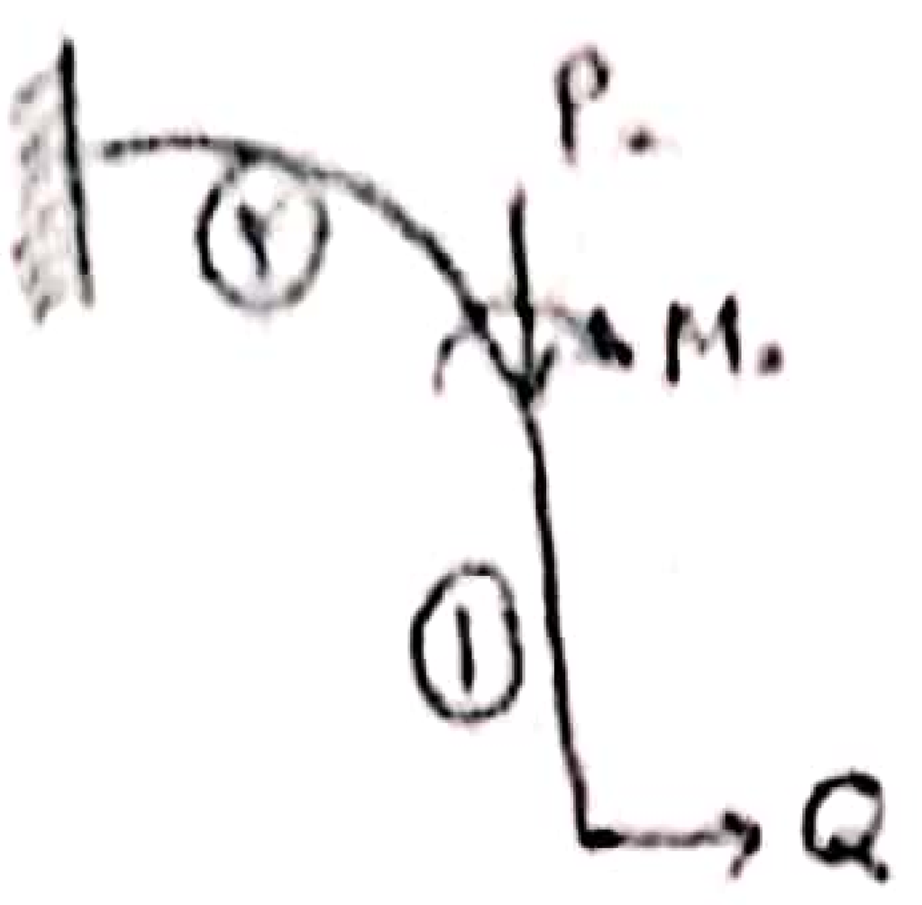
$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right|$ <
- 9.7 MPa Aluminum
- 12.1 MPa Steel

حالت دوم یعنی تماس استوانه با صفحه، بیشترین حالت تماس است.

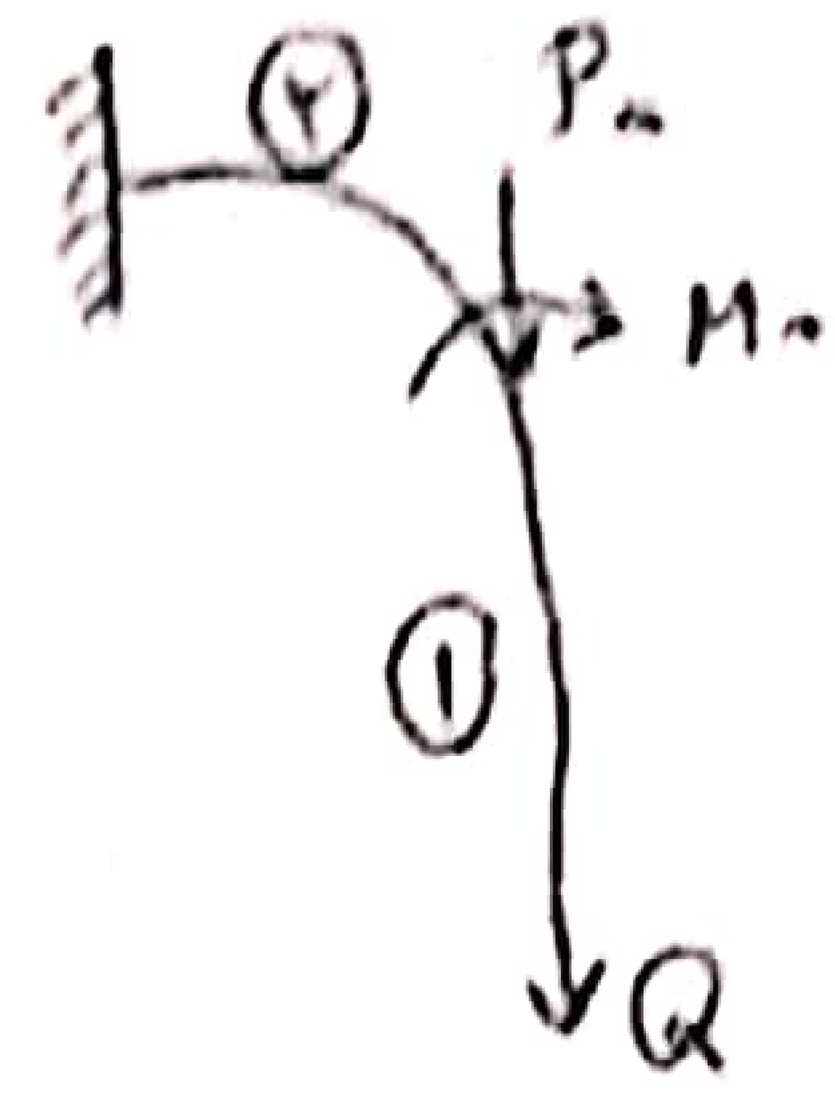
جواب سوال ۲)

در انتهای تیر باید دو نیروی افقی و عمودی مجازی وارد داد.

نیرو افقی



نیرو عمودی



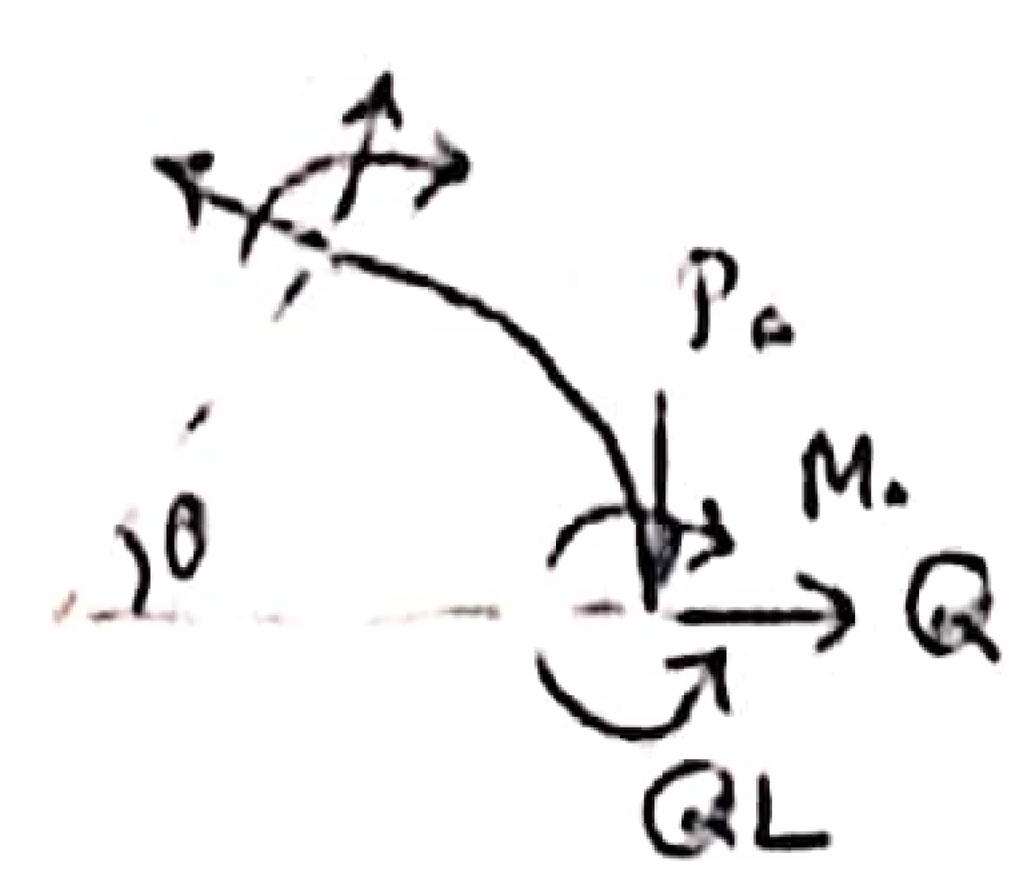
$$U = U_1 + U_2$$

نیرو افقی

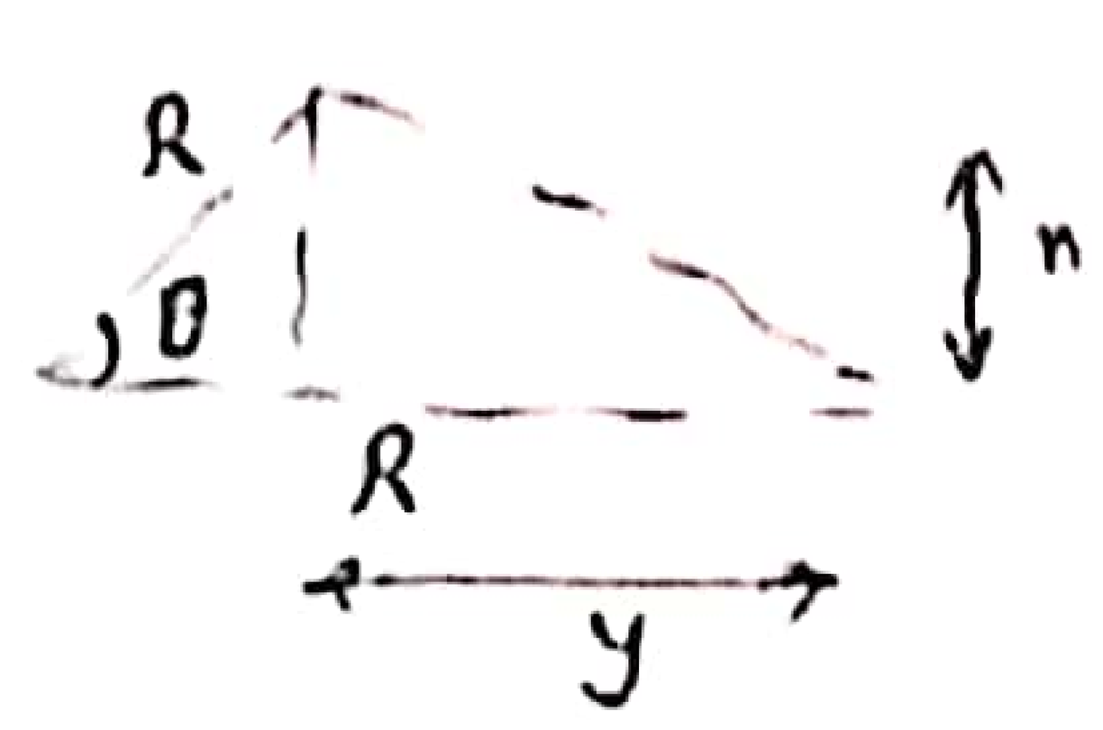
بخش ①

$$U_1 = \int_0^L \frac{M^2}{2EI_1}\,dx = \int_0^L \frac{(Qx)^2}{2EI_1}\,dx$$

بخش ②



$$U_2 = \int_0^{\pi/2} \frac{M^2\,d\theta}{2A_2eE} + \int_0^{\pi/2} \frac{F_\theta^2\,R\,d\theta}{2A_2E}$$



$$M = -M_0 + QL + QR\sin\theta - P_0(R - R\cos\theta)$$

$$F_\theta\cos\theta - F_r\sin\theta = Q$$

$$F_\theta\sin\theta + F_r\cos\theta = P_0$$

جمله اول در $\cos\theta$ و جمله دوم در $\sin\theta$ ضرب شده و با هم جمع شود.

$$\Rightarrow F_\theta = Q\cos\theta + P_0\sin\theta$$

$$\Rightarrow U_2 = \int_0^{\pi/2} \frac{\left(-M_0 + QL + QR\sin\theta - P_0(R - R\cos\theta)\right)^2 d\theta}{2A_2eE} + \int_0^{\pi/2} \frac{(Q\cos\theta + P_0\sin\theta)^2 R\,d\theta}{2A_2E}$$

$$U = U_1 + U_2 \longrightarrow \delta_x = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$\Rightarrow \delta_x = \left.\int_0^{\pi/2} \frac{\left(-M_0 + QL + QR\sin\theta - P_0(R - R\cos\theta)\right)(L + R\sin\theta)\,d\theta}{A_2eE}\right|_{Q=0} + \left.\int_0^{\pi/2} \frac{(Q\cos\theta + P_0\sin\theta)\cos\theta\,R\,d\theta}{A_2E}\right|_{Q=0}$$

$$+ \left.\int_0^L \frac{(Qx)\,x\,dx}{EI_1}\right|_{Q=0}$$

$$\Rightarrow \delta_x = \int_0^{\pi/2} \frac{(-M_0 - P_0(R - R\cos\theta))(L + R\sin\theta)\,d\theta}{A_2eE} + \int_0^{\pi/2} \frac{P_0\sin\theta\cos\theta\,R\,d\theta}{A_2E}$$

صورت انتگرال : $(-M_0 - P_0R + P_0R\cos\theta)(L + R\sin\theta) =$

$$-M_0L - M_0R\sin\theta - P_0RL - P_0R^2\sin\theta + P_0RL\cos\theta + P_0R^2\sin\theta\cos\theta =$$

$$-(M_0 + P_0R)L - (M_0R + P_0R^2)\sin\theta + P_0RL\cos\theta + P_0R^2\sin\theta\cos\theta$$

$$\Rightarrow \delta_x = \frac{1}{A_2eE}\left[-(M_0 + P_0R)L\theta + (M_0R + P_0R^2)\cos\theta + P_0RL\sin\theta + \frac{1}{2}P_0R^2\sin^2\theta\right]_0^{\pi/2} +$$

$$\frac{1}{A_2E}\left[\frac{1}{2}P_0R\sin^2\theta\right]_0^{\pi/2}$$

$$\Rightarrow \delta_x = \frac{1}{A_2eE}\left[-(M_0 + P_0R)L\frac{\pi}{2} + 0 + P_0RL + \frac{1}{2}P_0R^2 - 0 - (M_0R + P_0R^2) - 0 - 0\right] +$$

$$\frac{1}{A_2E}\left[\frac{1}{2}P_0R - 0\right]$$

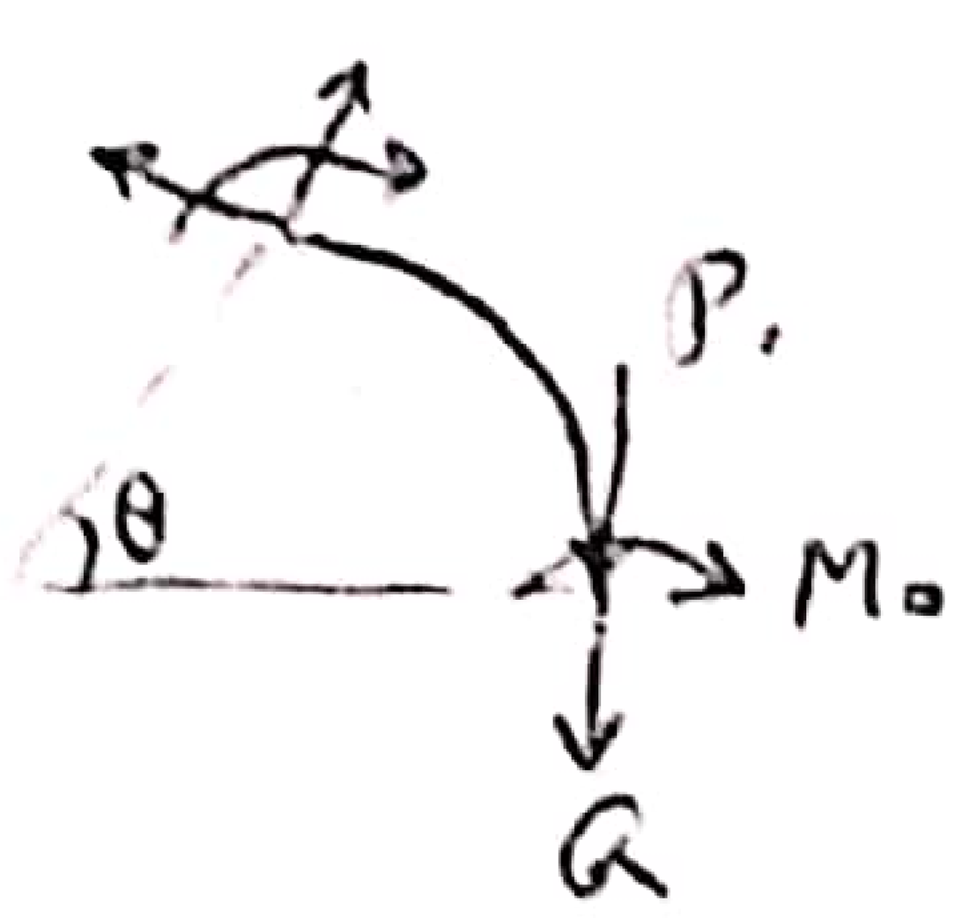
$$\Rightarrow \delta_x = \frac{1}{A_2eE}\left[-\frac{1}{2}(M_0 + P_0R)\pi L + P_0RL - M_0R - \frac{1}{2}P_0R^2\right] + \frac{1}{A_2E}\left[\frac{1}{2}P_0R\right] \quad (*)$$

نیروی عمودی

بخش ①

$$U_1 = \int_0^L \frac{F_1^2\,dx}{2A_1E} = \int_0^L \frac{Q^2\,dx}{2A_1E}$$

بخش ②



$$U_2 = \int_0^{\pi/2} \frac{M^2\,d\theta}{2A_2eE} + \int_0^{\pi/2} \frac{F_\theta^2\,R\,d\theta}{2A_2E}$$

$$M = -M_0 - (P_0 + Q)(R - R\cos\theta)$$

$$F_\theta\cos\theta - F_r\sin\theta = 0$$

$$F_\theta\sin\theta + F_r\cos\theta = P_0 + Q$$

جمله اول در $\cos\theta$ و جمله دوم در $\sin\theta$ ضرب شده و با هم جمع شود.

$$F_\theta = (P_0 + Q)\sin\theta$$

$$\Rightarrow U_2 = \int_0^{\pi/2} \frac{\left(-M_0 - (P_0 + Q)(R - R\cos\theta)\right)^2 d\theta}{2A_2eE} + \int_0^{\pi/2} \frac{(P_0 + Q)^2\sin^2\theta\,R\,d\theta}{2A_2E}$$

$$U = U_1 + U_2 \longrightarrow \delta_y = \frac{\partial U}{\partial Q}\Big|_{Q=0}$$

$$\Rightarrow \delta_y = \int_0^{\pi/2} \frac{(-M_0 - (P_0 + Q)(R - R\cos\theta))(R\cos\theta - R)\,d\theta}{A_2 e E}\Big|_{Q=0} + \int_0^{\pi/2} \frac{(P_0 + Q)\sin^2\theta\, R\,d\theta}{A_1 E}\Big|_{Q=0}$$

$$+ \int_0^{L} \frac{Q\,dx}{A_1 E}\Big|_{Q=0}$$

$$\Rightarrow \delta_y = \int_0^{\pi/2} \frac{(-M_0 - P_0(R - R\cos\theta))(R\cos\theta - R)\,d\theta}{A_2 e E} + \int_0^{\pi/2} \frac{P_0 \sin^2\theta\, R\,d\theta}{A_2 E}$$

بسط صورت :

$$-M_0 R\cos\theta + M_0 R - P_0 R^2\cos\theta + P_0 R^2 + P_0 R^2\cos^2\theta - P_0 R^2\cos\theta =$$

$$M_0 R + P_0 R^2 - (M_0 R + 2P_0 R^2)\cos\theta + P_0 R^2\cos^2\theta$$

$$\Rightarrow \delta_y = \frac{1}{A_2 e E}\left[(M_0 R + P_0 R^2)\theta - (M_0 R + 2P_0 R^2)\sin\theta + \frac{1}{2}P_0 R^2\left(\theta + \frac{1}{2}\sin 2\theta\right)\right]_0^{\pi/2} +$$

$$\frac{1}{A_2 E}\left[\frac{1}{2}P_0 R\left(\theta - \frac{1}{2}\sin 2\theta\right)\right]_0^{\pi/2}$$

$$\Rightarrow \delta_y = \frac{1}{A_2 e E}\left[(M_0 R + P_0 R^2)\frac{\pi}{2} - (M_0 R + 2P_0 R^2) + \frac{1}{2}P_0 R^2\left(\frac{\pi}{2} + 0\right) - 0 - 0 - 0\right] +$$

$$\frac{1}{A_2 E}\left[\frac{1}{2}P_0 R\left(\frac{\pi}{2} - 0\right) - 0\right]$$

$$\Rightarrow \delta_y = \frac{1}{A_2 e E}\left[\frac{1}{2}M_0 R\pi + \frac{3}{4}P_0 R^2\pi - M_0 R - 2P_0 R^2\right] + \frac{1}{A_2 E}\left[\frac{1}{4}P_0 R\pi\right] \quad (*)$$